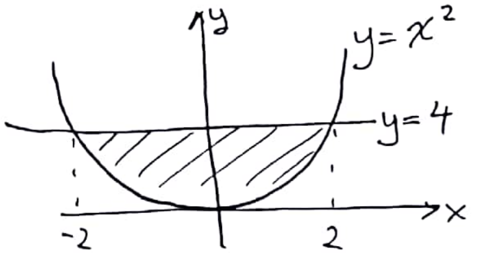
	Exam II	
Name:	Department: Comp. Eng.	GRADE
Student No:	Course: Calculus II	
Signature:	Date: 14/05/2018	

Demonstrate your solution steps clearly.

1. Find the volume of the solid bounded below by the graph of $z = x^2$ and above the region in the xy-plane enclosed by the curves $y = x^2$, $y = 4$.



$-2 \leq x \leq 2$

$x^2 \leq y \leq 4$

$$\text{Volume} = \int_{x=-2}^{2} \int_{y=x^2}^{4} x^2\, dy\, dx$$

$$= \int_{x=-2}^{2} x^2 y \Big|_{y=x^2}^{4} dx$$

$$= \int_{x=-2}^{2} x^2(4-x^2)dx$$

$$= \frac{4x^3}{3} - \frac{x^5}{5} \Big|_{-2}^{2}$$

$$\boxed{128/15}$$

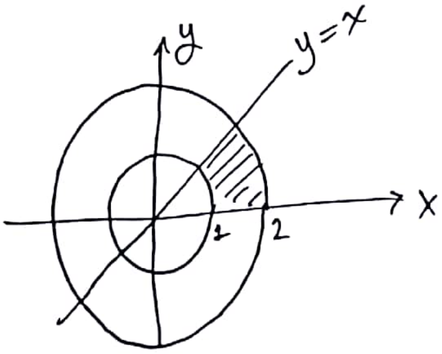
2. Use polar coordinates to set up and evaluate the double integral,

$$\iint_R \arctan\left(\frac{y}{x}\right) dA$$

where R is the region in the xy-plane bounded by $1 \leq x^2 + y^2 \leq 4$, $0 \leq y \leq x$.

In polar coordinates: $1 \leq r \leq 2$

$0 \leq \theta \leq \pi/4$



$$I = \iint_R \arctan\left(\frac{y}{x}\right) dA$$

$$= \int_{\theta=0}^{\pi/4} \int_{r=1}^{2} \arctan(\tan\theta)\, r\, dr\, d\theta$$

$$= \int_{\theta=0}^{\pi/4} \int_{r=1}^{2} \theta r\, dr\, d\theta = \int_{\theta=0}^{\pi/4} \theta \frac{r^2}{2} \Big|_{r=1}^{2} d\theta$$

$$= \frac{3}{2} \int_{\theta=0}^{\pi/4} \theta\, d\theta = \frac{3}{2} \frac{\theta^2}{2} \Big|_{\theta=0}^{\pi/4}$$

$$\boxed{3\pi^2/64}$$

3. Use chain rule to find $\frac{\partial w}{\partial s}$ if $w = xe^y - yz$ with $x = -s + t^2$, $y = (t+s)^2$ and $z = \ln(t+s)$ when $t = 0$, $s = 1$ (No credits will be given to direct calculation).

When $t=0,\ s=1 \Rightarrow x=-1,\ y=1,\ z=0$

$$\frac{\partial w}{\partial s} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial s} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial s}$$

$$= e^y(-1) + (xe^y - z)\,2(t+s) - y\,\frac{1}{t+s}$$

$$\left.\frac{\partial w}{\partial s}\right|_{\substack{t=0\\ s=1}} = -e - 2e - 1$$

$$\boxed{-3e-1}$$

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4. Find local extrema and saddle points of $f(x, y) = x^3 - 3x + 2y^2 - 8y + 11$.

$f_x = 3x^2 - 3 = 0 \Rightarrow x = \mp 1.$

$f_y = 4y - 8 = 0 \Rightarrow y = 2.$

Critical points: $(-1,2), (1,2)$

$f_{xx} = 6x$

$f_{xy} = 0$

$f_{yy} = 4$

At $(-1,2)$:

$\Delta = f_{xx}f_{yy} - f_{xy}^2 = -6\cdot 4 < 0$

$\boxed{(-1,2) \to \text{saddle point}}$

At $(1,2)$:

$\Delta = 24 > 0,\ \underbrace{f_{xx}}_{=6} > 0$

$\boxed{(1,2) \to \text{local minimum}}$

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5. Let $T(x, y, z) = \dfrac{360}{\sqrt{x^2 + y^2 + z^2}}$. Find the directional derivative of T at $P(1,2,2)$ in the direction toward the point $Q(2,1,3)$.

$$\frac{\partial T}{\partial x} = 360 \cdot \left(\frac{-1}{2}\right)\frac{2x}{(x^2+y^2+z^2)^{3/2}} = \frac{-360}{(x^2+y^2+z^2)^{3/2}}\,x$$

$$\nabla T = \frac{-360}{(x^2+y^2+z^2)^{3/2}}(x\hat{i} + y\hat{j} + z k)$$

$$\nabla T(1,2,2) = \frac{-360}{27}(\hat{i} + 2\hat{j} + 2k)$$

$\vec{PQ} = \hat{i} - j + k \qquad \vec{u} = \dfrac{\vec{PQ}}{|\vec{PQ}|} = \dfrac{1}{\sqrt{3}}(i - j + k)$

$\nabla T \cdot \vec{u} =$

$\dfrac{-360}{27}(i + 2j + 2k)\cdot\dfrac{1}{\sqrt{3}}(i - j + k)$

$= \dfrac{-40}{3\sqrt{3}}(1 - 2 + 2)$

$$\boxed{-40/3\sqrt{3}}$$